A Game-Theoretic Approach for Pricing and Determining Quality Levels of Cybersecurity Products Under an Exogenous Information-sharing Program

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Abstract. Price and quality level of products are two important decisions of any business. This paper provides equilibrium solutions for these decisions of two players for a cybersecurity ecosystem, including a solution provider and an information provider. We assume that end users join a cybersecurity ecosystem based on the prices and qualities of the solution provider's and information provider's products; so that, the increasing of the prices or/and decreasing the qualities will reduce the number of end-users of that ecosystem. Also, it is assumed that there is an exogenous information-sharing program under which the effect of quality of the information provider's product on the cybersecurity level depends on the level of information-sharing. Under this information-sharing program, the equilibrium solutions of prices and qualities of these two players are given. Also, some results and insights are given with a numerical example.

Keywords: Cybersecurity ecosystem; pricing; quality level; information sharing; game theory.

1 Introduction

The enterprise information systems security directly affects other firms, and the interconnectedness of information assets inevitably affects the choice of strategy of information security. Also, information-sharing (InSh) is a strategic complementary relationship to information security investments. By InSh between companies, companies can reduce the security cost and improve the information security level [1]. Therefore, companies can benefit from InSh [2-4]. However, with the expansion of information and communication technology, the InSh efficiency is not fully understood [5]; Therefore, it is very important to develop economic models in which InSh and product quality are considered together. For details of literature refer to [1, 5-10]. To our knowledge, there is no research that has formulated a security ecosystem issue from a supply chain perspective. The under-hand paper tries to find the equilibrium solutions of two important players of a cybersecurity ecosystem including a solution provider and an information provider. Due to the nature of the problem, game theory has been used to find equilibrium solutions. The research questions of this study are as follows. ***RQ 1.*** *What are the equilibrium prices and qualities under the InSh program?* ***RQ 2.*** *How do parameters affect player profits?* ***RQ 3.*** *Are there any conditions under which the players' equilibrium profits improve significantly?*

2 Problem description

Parameters

υ	A fixed value in the function of number of end-users (the number of end-users when the security level and prices are close to zero)
α	Demand sensitivity to the security level
γ	Demand sensitivity to the price level
c_S	Cost of the solution provider's product
c_I	Cost of the information provider's product
θ_S	Coefficient of the effect of the quality level of the solution provider's product on the solution provider costs
θ_I	Coefficient of the effect of the quality level of the information provider's product on the information provider costs
ψ	Information-sharing level

Decision variables

p_S	Price of the solution provider's product
p_I	Price of the information provider's product
q_S	Quality level of the solution provider's product
q_I	Quality level of the information provider's product

Demand and profit functions

U	Number of installed-base or end-users
$\mathcal{L}$	The level of cybersecurity
π_S	Profit function of the solution provider
π_I	Profit function of the information provider

Consider a solution provider and an information provider in a cybersecurity ecosystem, each of which wants to determine the price and quality of its product. For brevity, we refer to [11] for definitions and roles of cybersecurity ecosystem's agents. The number of customers of this ecosystem is a function of the prices and qualities of the solution provider's and information provider's products. Increasing the quality increases the number of customers and increasing the price reduces the number of customers. The profit of these two players is a function of price, cost of each product unit, and costs of increasing product quality. Due to the frequent use of linear functions in the related literature, the linear functions are used to formulate the effect of price and quality on the number of end-users of this ecosystem. Therefore, according to the defined notations and the above explanations, the number of customers of this ecosystem is calculated by Equation (1). The cybersecurity level and the profit functions of two considered players are provided in Equations (2)-(4), respectively. The demand for cybersecurity products increases with the increase in the security level of that product and the decrease in its price. This issue is formulated in Equation (1). The level of cybersecurity increases by increasing the quality of the solution and information providers (Equation (2)) but this effect may not be the same in general; so, we added ψ to model different effects. Equations (3)-(4) show the profits, which are equal to revenue minus quality cost. In this research, we look for interior solutions, so all the functions should be considered positive.

$$U = \upsilon + \alpha\mathcal{L} - \gamma(p_S + p_I) \quad (1) \qquad \mathcal{L} = q_S + \psi q_I \quad (2)$$

$$\pi_S = (p_S - c_S)U - \theta_S q_S^2 \quad (3) \qquad \pi_I = (p_I - c_I)U - \theta_I q_I^2 \quad (4)$$

Game theory is widely used in the literature to analyze a wide range of multi-player decision problems. The equilibrium solutions of decision variables of the solution provider and the information provider are provided in ***Theorem 1*** based on ***Lemma 1***.

Lemma 1. π_S on p_S and q_S and π_I on p_I and q_I are jointly concave functions if $\alpha < 2\sqrt{\gamma\theta_S}$ and $\alpha < \frac{2}{\psi}\sqrt{\gamma\theta_I}$, respectively.

Theorem 1. *The equilibrium solutions of prices and qualities of the two players are provide in Equations (5)-(8). The necessary conditions were provided in Lemma 1.*

$$p_S^{NE} = c_S + \frac{2\theta_I\theta_S(\upsilon - (c_I + c_S)\gamma)}{6\gamma\theta_I\theta_S - \alpha^2(\theta_I + \theta_S\psi^2)} \quad (5) \qquad q_S^{NE} = \frac{\alpha\theta_I(\upsilon - (c_I + c_S)\gamma)}{6\gamma\theta_I\theta_S - \alpha^2(\theta_I + \theta_S\psi^2)} \quad (6)$$

$$p_I^{NE} = c_I + \frac{2\theta_I\theta_S(\upsilon - (c_I + c_S)\gamma)}{6\gamma\theta_I\theta_S - \alpha^2(\theta_I + \theta_S\psi^2)} \quad (7) \qquad q_I^{NE} = \frac{\alpha\theta_S(\upsilon - (c_I + c_S)\gamma)\psi}{6\gamma\theta_I\theta_S - \alpha^2(\theta_I + \theta_S\psi^2)} \quad (8)$$

Corollary 1. *Equilibrium prices and quality of solution provider and information provider's products have the following relationships with each other (See Equations (9) and (10)). Therefore:*

- *According to Equation (9), the marginal profits of two considered players are the same and is equal to* $\frac{2\theta_I\theta_S(\upsilon-(c_I+c_S)\gamma)}{6\gamma\theta_I\theta_S-\alpha^2(\theta_I+\theta_S\psi^2)}$.
- *According to Equation (10), for each specific value of the InSh level, the ratio of quality of solution provider's product to quality of information provider's product varies by a factor of* θ_S/θ_I*. This relationship also shows that with the increasing of the level of InSh or/and increasing the quality cost coefficient of solution provider's product or decreasing the quality cost coefficient of information provider's product, the difference between the qualities of two products increases.*

$$p_S^{NE} - c_S = p_I^{NE} - c_I = \frac{2\theta_I\theta_S(\upsilon - (c_I + c_S)\gamma)}{6\gamma\theta_I\theta_S - \alpha^2(\theta_I + \theta_S\psi^2)} \quad (9)$$

$$\frac{q_I^{NE}}{q_S^{NE}} = \frac{\theta_S}{\theta_I}\psi \quad (10)$$

3 Summary

This article presents the equilibrium solutions of two players in cybersecurity enhancement, including the solution provider and the information provider. Equilibrium solutions of prices and qualities of products are provided for these two players. This article can be expanded to consider other aspects of cybersecurity, such as corporate awareness and the role of government in improving security.

Acknowledgments

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Abstract

Price and quality level of products are two important decisions of any business. This paper provides equilibrium solutions for these decisions of two players for a cybersecurity ecosystem, including a solution provider and an information provider. We assume that end users join a cybersecurity ecosystem based on the prices and qualities of the solution provider's and information provider's products; so that, the increasing of the prices or/and **decreasing the qualities will reduce the number of end-users of that ecosystem. Also, it is assumed that there is an exogenous information-sharing program under which the effect of quality of the information** provider's product on the cybersecurity level depends on the level of **information-sharing. Under this information-sharing program, the equilibrium solutions of prices and qualities of these two players are given. Also, some results and insights are given with a numerical example.**

INTRODUCTION

The enterprise information systems security directly affects other firms, and the interconnectedness of information assets inevitably affects the choice of strategy of information security. Also, information-sharing (InSh) is a strategic complementary relationship to information security investments. By InSh between companies, companies can reduce the security cost and improve the information security level [1]. Therefore, companies can benefit from InSh [2-4]. However, with the expansion of information and communication technology, the InSh efficiency is not fully understood [5]; thus, promoting companies to share information is very important [5]. On the other hand, **insights** of **cybersecurity economics help decision-makers to make decisions that improve the management of situations that with catastrophic consequences and threaten the sustainability of the digital ecosystems [6].**

To our knowledge, there is no research that has formulated a security ecosystem issue from a supply chain perspective. A view in which the number of users is a function of the security level and the prices of security ecosystem products. The under-hand paper, try to find the equilibrium solutions of two important players of a cybersecurity ecosystem including a solution provider and an information provider. These players want to determine the price and quality level of their products, separately. We assume that end-users join this ecosystem according to the price level and product quality, so the number of end-users is a function of the price level and product quality level of products of this ecosystem. It is also assumed that as the level of InSh increases, the effect of the product quality of the information provider on the cybersecurity level increases. Due to the nature of the problem, game theory has been used to find equilibrium solutions. This paper examines this issue for the first time. The research questions of this study are as follows.

RQ 1. What are the equilibrium prices and qualities under the InSh program?
RQ 2. How do parameters affect player profits?
RQ 3. Are there any conditions under which the players' equilibrium profits improve significantly?

Problem Description

Consider a solution provider and an information provider in a cyberspace ecosystem, each of which wants to determine the price and quality of its product. The number of customers of this ecosystem is a function of the prices and qualities of the solution provider's and information provider's **products. Increasing the quality increases the number of customers and increasing the price reduces the number of customers. The profit of these two players is a function of price, cost of each product unit, and costs of increasing product quality. Due to the frequent use of linear functions in the related literature, the linear functions are used to formulate the effect of price and quality on the number of end-users of this ecosystem. Therefore, according to the defined notations and the above explanations, the number of customers of this ecosystem is calculated as follows.**

$$U = v + \alpha\mathcal{L} - \gamma(p_S + p_I) \tag{1}$$

$$\mathcal{L} = q_S + \psi q_I \tag{2}$$

$$\pi_S = (p_S - c_S)U^{NC} - \theta_S q_S^2 \tag{3}$$

$$\pi_I = (p_I - c_I)U^{NC} - \theta_I q_I^2 \tag{4}$$

Equilibrium solutions and analytical results

Game theory is widely used in the literature to analyze a wide range of multi-player decision problems. The equilibrium solutions of decision variables of the solution provider and the information provider are provided in Theorem 1 based on Lemma 1.

Lemma 1.

π_S on p_S and q_S and π_I on p_I and q_I are jointly concave functions.

Proof. The π_S' Hessian matrix is calculated as $\begin{pmatrix} -2\gamma & \alpha \\ \alpha & -2\theta_S \end{pmatrix}$, **which is negative definite, considering $\alpha < 2\sqrt{\gamma\theta_S}$. The condition for negative definite of π_I is $\alpha < \frac{2}{\psi}\sqrt{\gamma\theta_I}$. □**

Based on the Lemma 1, for concavity of profit functions, we consider $\alpha < 2\sqrt{\gamma\theta_S}\, Min\left\{1, \frac{1}{\psi}\right\}$.

Theorem 1.

The equilibrium solutions of prices and qualities of the two players are provide in below. The necessary conditions were provided in Lemma 1.

$$p_S^{NE} = c_S + \frac{2\theta_I\theta_S(v - (c_I + c_S)\gamma)}{6\gamma\theta_I\theta_S - \alpha^2(\theta_I + \theta_S\psi^2)} \tag{5}$$

$$q_S^{NE} = \frac{\alpha\theta_I(v - (c_I + c_S)\gamma)}{6\gamma\theta_I\theta_S - \alpha^2(\theta_I + \theta_S\psi^2)} \tag{6}$$

$$p_I^{NE} = c_I + \frac{2\theta_I\theta_S(v - (c_I + c_S)\gamma)}{6\gamma\theta_I\theta_S - \alpha^2(\theta_I + \theta_S\psi^2)} \tag{7}$$

$$q_I^{NE} = \frac{\alpha\theta_S(v - (c_I + c_S)\gamma)\psi}{6\gamma\theta_I\theta_S - \alpha^2(\theta_I + \theta_S\psi^2)} \tag{8}$$

Theorem 2.

The equilibrium number of end-users and the equilibrium profits of the solution provider and the information provider are obtained below under the conditions that were provided in Lemma 1.

$$U^{NE} = \frac{2\gamma\theta_I\theta_S((c_I + c_S)\gamma - v)}{\alpha^2(\theta_I + \theta_S\psi^2) - 6\gamma\theta_I\theta_S} \tag{9}$$

$$\pi_S^{NE} = \frac{\theta_S(-\alpha^2 + 4\gamma\theta_S)((c_I + c_S)\gamma\theta_I - \theta_I v)^2}{(\alpha^2(\theta_I + \theta_S\psi^2) - 6\gamma\theta_I\theta_S)^2} \tag{10}$$

$$\pi_I^{NE} = \frac{\theta_I((c_I + c_S)\gamma\theta_S - \theta_S v)^2(4\gamma\theta_I - \alpha^2\psi^2)}{(\alpha^2(\theta_I + \theta_S\psi^2) - 6\gamma\theta_I\theta_S)^2} \tag{11}$$

Corollary 1

Equilibrium prices and quality of solution provider and information provider's products have the following relationships with each other. Therefore:

- *The marginal profits of two considered players are the same and is equal to $\frac{2\theta_I\theta_S(v-(c_I+c_S)\gamma)}{6\gamma\theta_I\theta_S-\alpha^2(\theta_I+\theta_S\psi^2)}$.*
- *For each specific value of the InSh level, the ratio of quality of solution provider's product to quality of information provider's product varies by a factor of θ_S/θ_I. This relationship also shows that with the increasing of the level of InSh or/and increasing the quality cost coefficient of solution provider's product or decreasing the quality cost coefficient of information provider's product, the difference between the qualities of two products increases.*

$$p_S^{NE} - c_S = p_I^{NE} - c_I = \frac{2\theta_I\theta_S(v - (c_I + c_S)\gamma)}{6\gamma\theta_I\theta_S - \alpha^2(\theta_I + \theta_S\psi^2)} \tag{12}$$

$$\frac{q_I^{NE}}{q_S^{NE}} = \frac{\theta_S}{\theta_I}\psi \tag{13}$$

Numerical Example

In this section, by presenting a numerical example, the results and management tips are presented. Figures 3 and 4 show the behavior of the two players functions in terms of a number of parameters and decision variables. Based on the results, the following points can be expressed.

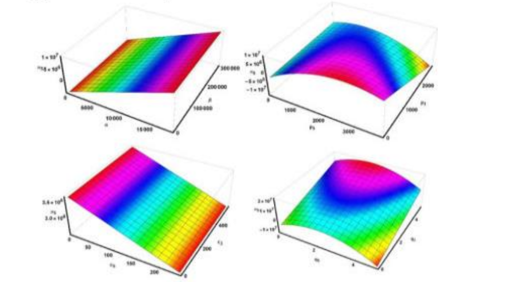

Figure 3. The profit function of the solution provider with respect to the some parameters and variables

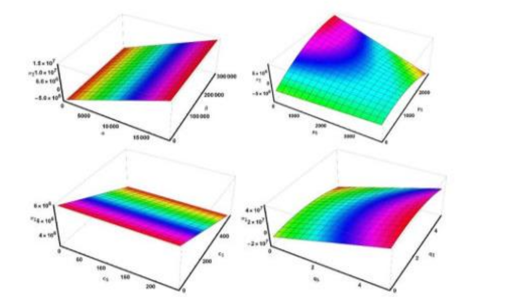

Figure 4. The profit function of the information provider with respect to the some parameters and variables

Figure 1 shows that the increasing of the demand elasticity with respect to the security level and InSh level increases the equilibrium prices, qualities and profits of both players. According to those figures and necessary conditions that were provided in Lemma 1, it can be said that the increasing of prices, qualities and equilibrium profits at the border of $\alpha < 2\sqrt{\gamma\theta_S}\, Min\left\{1, \frac{1}{\psi}\right\}$ is accompanied by a steep slope. So if players can influence the "demand elasticity with respect to the security level" and/or "InSh level" parameters, they get the **most benefit when** they try to adjust the parameters so that the α is closer to the $2\sqrt{\gamma\theta_S}\, Min\left\{1, \frac{1}{\psi}\right\}$.

Figure 2 show the increasing of the production costs of the solution provider has a direct (positive) effect on the equilibrium price of the solution provider's **product and a non-directional (negative) effect on the equilibrium price of the** information provider's product. In other words, increasing the costs of the **solution provider increases** the equilibrium price of the solution provider's **product and decreases the equilibrium price of the information provider. There is also the same behavior for the costs of the information provider.**

It can also be seen that increasing the costs of the solution provider's product or information provider's product has a **non-directional (negative) effect on the** equilibrium quality of the solution provider's product as well as on the equilibrium quality of the information provider's **product. In other words, increasing the costs of each player reduces the equilibrium qualities of both players products.**

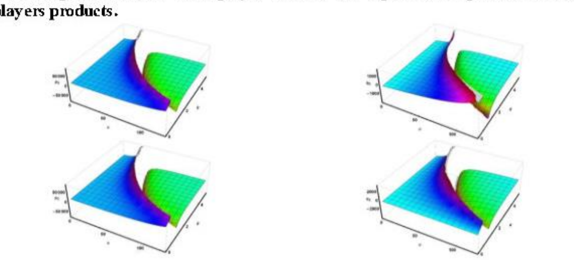

Figure 1. The effect of the demand elasticity on equilibrium decision variables and equilibrium profits with respect to the security level and InSh level

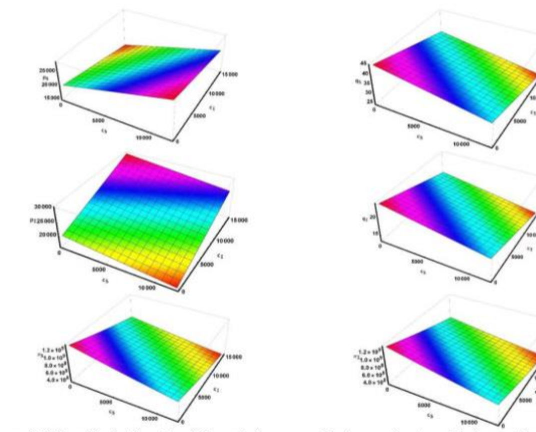

Figure 2. The effect of costs of the solution provider's product and information provider's product on equilibrium decision variables and equilibrium profits

Summary

With the increasing use of the Internet, especially after Corona, the security of cyberspace is one of the most important challenges of today. Cyberspace security is an issue that affects most people in society, from governments to the private sector and the public. Researchers also consider InSh as one of the strategies to increase cyberspace security. Therefore, research in the field of InSh is very important to strengthen cyberspace security. This article presents the equilibrium solutions of two players in cyberspace security enhancement, including the solution provider and the information provider. Equilibrium solutions of prices and qualities of products are provided for these two players. Results and insights are presented with a numerical example. In this paper, a boundary for the parameters is found so that the players around the boundary get the most benefit from the equilibrium solutions. This article can be expanded to consider other aspects of cybersecurity, such as corporate awareness and the role of government in improving security. Other problem-solving approaches such as evolutionary games and dynamic games can also improve this research.

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